Your Signature ______________________

1.(a) (4 points) Use the 64-bit long real format to find the decimal equivalent of the following floating-point machine number

1 10000001010 10010011000000000000000000000000000000000000000000000

1.(b) (8 points) using the below R-code, we are interested in computing the

$$\int_0^1 e^{-x^2} dx$$

to an error of at most $\frac{1}{2} \times 10^{-4}$.

```
> Approxintegrate = function(f,a,b,n){
+ m = n + 1
+ h = (b-a)/(m-1)
+ x = seq(a,b, by= h)
+ y = f(x)
+ I = h * (0.5*(y[1]) + sum(y[2:m-1]) + 0.5*(y[m]) )
+ I}
```

With justification write R-commands which uses the above code to give you desired answer.

1.(c) (8 points) Explain the below R-code (in each case describing what the command intends and the resulting output)

```
> .Machine$double.xmin

[1] 2.225074e-308

> .Machine$double.xmin/2

[1] 1.112537e-308

> bitsOfPrecision = function(x)max(which( x !=  x*(1+2^-(1:60))))
> bitsOfPrecision(.Machine$double.xmin/2)

[1] 51
```

Your Signature ____________________

2. (a) Let $f, g : \mathbb{N} \to (0, \infty)$

 (i) (2 points) Define what it means to say $f(n) = o(g(n))$.

 (ii) (8 points) Suppose $f(n) = \frac{4^n n^5}{n!}$ and $g(n) = n^{-0.9n}$. Is $f(n) = o(g(n))$?

2.(b) One of the two software packages, R or MATLAB should be chosen to process data collections. Average processing time of the package R for n records is given by $R(n) = 0.001n$ milliseconds and the average processing time of the package MATLAB for n records is given by $M(n) = 500\sqrt{n}$ milliseconds.

 (i) (2 points) Which package has better performance (w.r.t processing times) in the Big-O sense?

 (ii) (3 points) Suppose the data collections each contain up to 10^9 records. If you wish to use the package that performs better then which one would you choose ?

3. (10 points) Suppose $y_0 = 0$, $y_1 = 1$ and

$$y_{n+1} = y_n + \frac{(2 - e^{y_n})(y_n - y_{n+1})}{e^{y_n} - e^{y_{n+1}}}$$

for $n \geq 2$. Decide, with appropriate justification, if y_n converges to an $y \in \mathbb{R}$. If it does then find y and the order of convergence.

4. Consider $f : [1, 2] \to \mathbb{R}$ given by

$$f(x) = \frac{1}{x},$$

for $x \in [1, 2]$. Let P be the polynomial of degree less than or equal to n that interpolates the points $\{(x_i, f(x_i) : 0 \leq i \leq n\}$. Let $e : [1, 2] \to \mathbb{R}$ be given by

$$e(x) = f(x) - p(x).$$

for $x \in [1, 2]$.

 (a) (5 points) Show that $| \, e(x) \, | \leq 1$ for all $x \in [1, 2]$

 (b) (10 points) With justification show that there exists a choice of $n, x_0, x_1, \ldots, x_n$ such that $| \, e(x) \, | < 10^{-3}$ for all $x \in [1, 2]$?